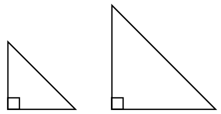
Similarity and Congruence

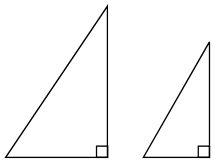
Geometric figures that are exactly the same shape, but not necessarily the same size, are called **similar figures (~)**. In *similar figures*, all the pairs of **corresponding angles** are the same measure, and all the pairs of **corresponding sides** are in the same ratio. This ratio, in its reduced form, is called the **scale factor**. When all pairs of *corresponding sides* are in the same ratio as the *scale factor*, we say that the **sides** are in proportion.

Some geometric figures are always similar.

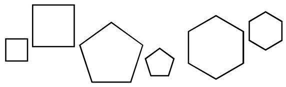
1. All **triangles** whose **angles' (∠)** measures of **degree (°)** are 45°, 45°, and 90° are similar to each other.



2. All *triangles* whose *angles' (∠)* measures of *degree* (°) are 30°, 60°, and 90° are similar to each other.



3. All **regular polygons** with the same number of *sides* are similar to each other.



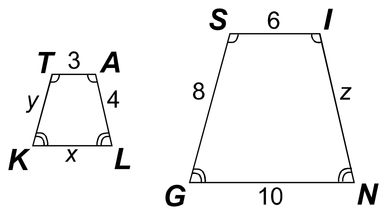
Remember: A *regular polygon* is a **polygon** that is **equilateral** and **equiangular**. Therefore, all its sides are **congruent (≅)** and all angles are *congruent (≅)*.

Note: Circles seem to be similar, but since they have no angle measures, we don't include them in this group.

Using Proportions Geometrically

If we know two shapes are similar, and we know some of the lengths, we often can find some of the other measures of those shapes. Look at the two similar figures below. We have labeled the trapezoids *TALK* and *SING*.

Trapezoids *TALK* and *SING*



By locating the corresponding angles, we can say that

Trapezoid *TALK* ~ **Trapezoid** *SING*.

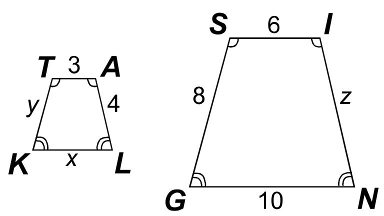
Note: ~ is the symbol for similar.

To find the values of x, y, and z, we must first find a pair of corresponding sides with lengths given.

- Side *TA* and side *SI* are a pair of corresponding sides.
- It is given that $TA = 3$ and $SI = 6$.
- So, we can set up a ratio $\frac{TA}{SI} = \frac{3}{6}$.
- When we reduce the ratio, we get the scale factor, which is $\frac{1}{2}$.
- This means that every length in *TALK* is one-half the **corresponding** length in *SING*.

Now we can use the scale factor to make proportions and find x, y, and z. Remember to be consistent as you set up the proportions. Since my scale factor was determined by a comparison of *TALK* to *SING*, I will continue in that order: ($\frac{TALK}{SING}$).

Trapezoids *TALK* and *SING*



$\frac{1}{2} = \frac{x}{10}$	$\frac{1}{2} = \frac{y}{8}$	$\frac{1}{2} = \frac{4}{z}$
$2x = 10$	$2y = 8$	$8 = 1z$
$x = 5$	$y = 4$	$8 = z$

What is the **perimeter (*P*)**, or distance around the *polygon*, of *TALK*?

Did you get 16?

Can you guess the *perimeter* of *SING*?

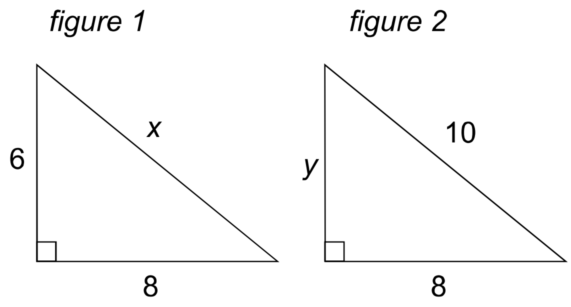
If you guessed 32, you are correct.

Does it make sense that the perimeters should be in the same ratio as the scale factor?

Yes, because the perimeters of *TALK* and *SING* are corresponding lengths. In addition, all *corresponding* lengths in similar figures are in proportion!

Using Proportions to Find Heights

Look at the figures below. They are from number 8 in the previous practice.



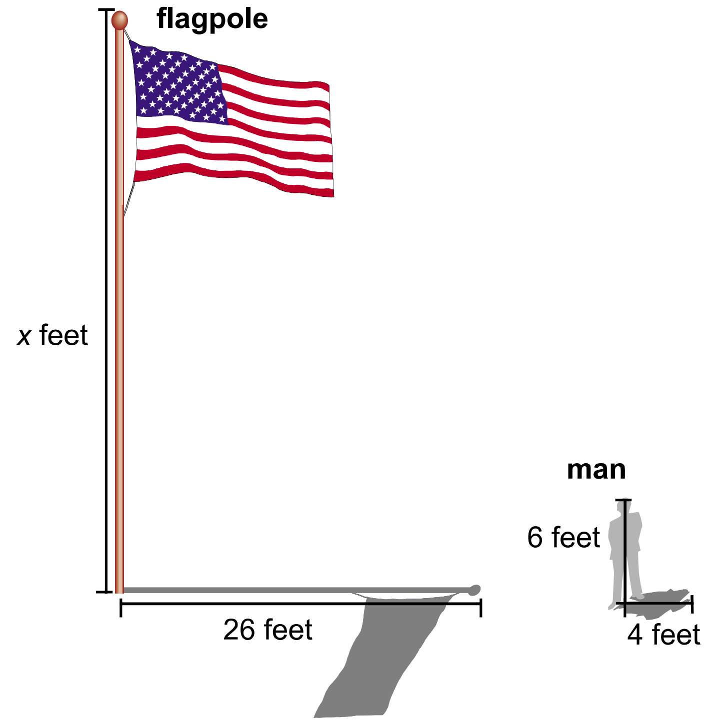
Here is what we know about *figure 1* and *figure 2* above.

- Their scale factor is $\frac{1}{1}$. This makes all the pairs of corresponding sides the same length.

- We already knew that their corresponding angles were the same measure because we knew that they were similar. This makes the triangles identical to each other.

Geometric figures that are *exactly* the same *shape* and *exactly* the same *size* are *congruent* to each other. The symbol for congruence, $\cong$, is a lot like the symbol for similar, but the equal sign, =, underneath it tells us that two things are *exactly* the same *size*.

We can use proportions to find the lengths of some items that would be difficult to measure. For instance, if we needed to know the height of a flagpole without having to inch our way up, we could use proportions. See the example on the following page.



A 6-foot man casts a 4-foot shadow at the same time a flagpole casts a 26-foot shadow. Find the **height (*h*)** of the flagpole.

To solve a problem like this, set up a proportion comparing corresponding parts.

$$\frac{\text{man's height}}{\text{man's shadow}} = \frac{\text{flagpole's height}}{\text{flagpole's shadow}}$$

$$\frac{6}{4} = \frac{x}{26}$$ ← cross multiply

$$4x = 6 \text{ x } 26$$

$$4x = 156$$ ← divide both sides by 4

$$\frac{4x}{4} = \frac{156}{4}$$

$$x = 39 \text{ feet}$$

Now try the following practice.